Worksheet 9 - Rejection regions $+$ Power

Monday, February 23, 2026

Math 58B - Jo Hardin

Name: _____

Names of people you worked with: _____

Where do you like to study? Do you have a go-to place or do you prefer different spots?

Task: You are trying to convince your college to offer more vegetarian options at mealtime, and you believe that 40% of students are vegetarians. The college says they disagree, and they won't change the offerings unless you can convince them that more than 35% of the student body is vegetarian.

Let's say it is a one sided test with level of discernibility of 0.1. Also, assume that the sample size will be big enough so that the central limit theorem holds. Start by suggesting a sample size of 50 people.

   a. What are the null and alternative hypotheses?
   b. What is the formula for the Z score which will assess whether or not you reject H_0?
   c. What is the Z^* value above which an observed Z score will reject H_0? (Use R.)
   d. What $\hat{p}$ do you need to get to reject H_0 (with n = 50)?
   e. If, in fact, the true proportion of vegetarians is $p = 0.4$, what is the probability that you will reject? (Again, if you can draw the correct picture, I'll tell you the number.)
   f. What is your power at $p = 0.4$?
   g. What would you do here to increase your power? Is your solution always feasible in other experiments?

Solution:
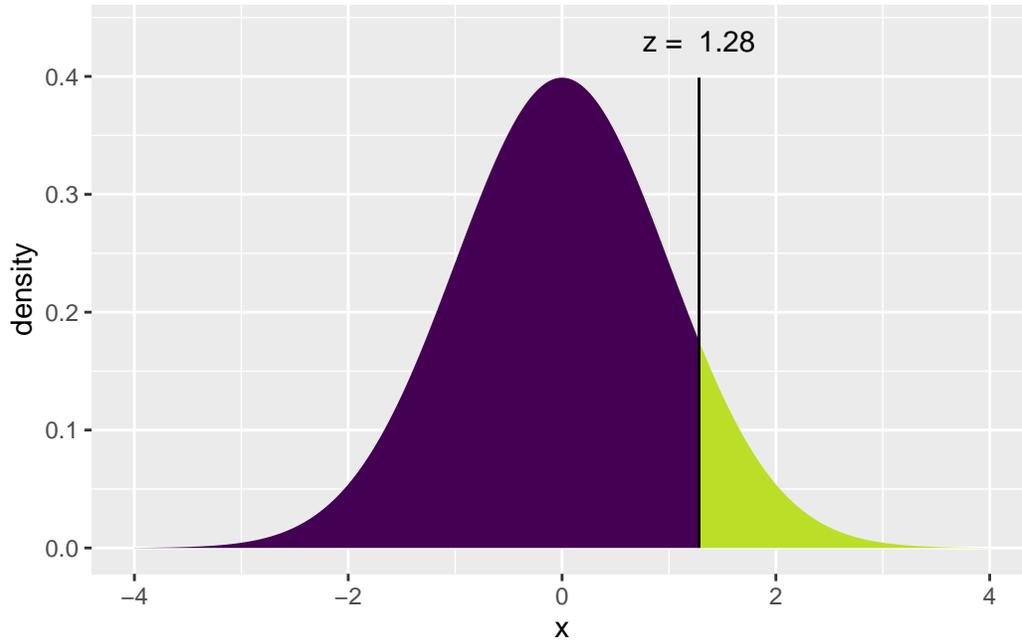
a. $H_0 : p = 0.35$, $H_a : p > 0.35$

b.
$$Z = \frac{\hat{p} - p}{\sqrt{p \cdot (1-p)/n}} = \frac{\hat{p} - 0.35}{\sqrt{0.35 \cdot 0.65/50}}$$
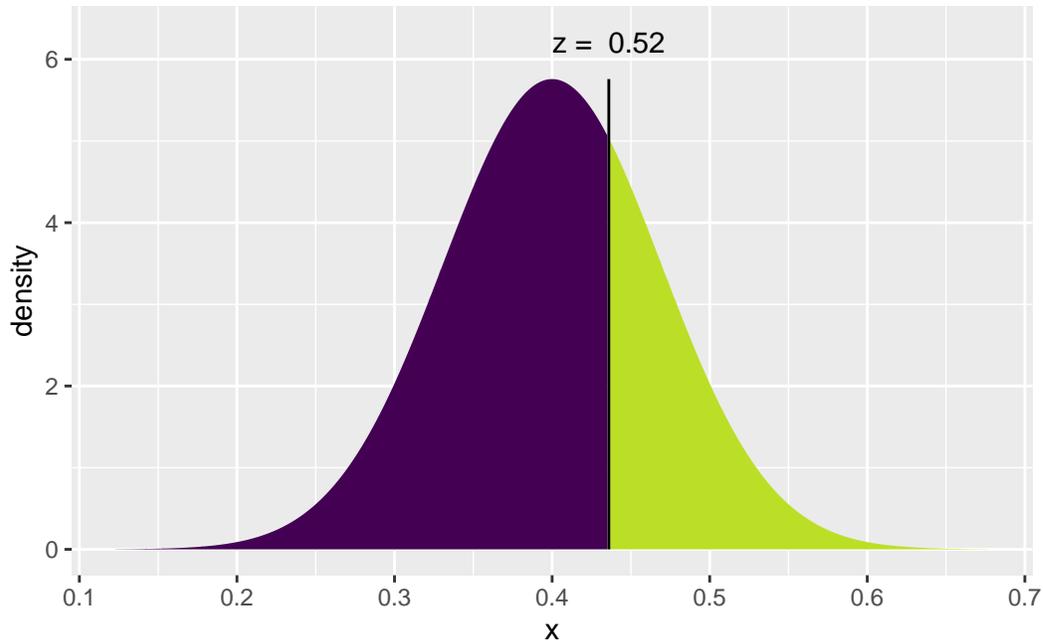
c.

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xqnorm(0.9, mean = 0, sd = 1)
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```
[1] 1.281552
```

d.
$$\frac{\hat{p} - 0.35}{\sqrt{0.35 \cdot 0.65/50}} > 1.28 \rightarrow \hat{p} > 0.436$$

e.

```
1 - xpnorm(0.436, mean = 0.4, sd = sqrt(0.4*0.6/50))
```

```
[1] 0.3016659
```

    f. With 50 observations, there is only a 0.3 probability (power) that the random sample would reject H_0 **even if** 40% of students were truly vegetarians.

    g. To get to a higher power, more observations need to be sampled. That isn't always possible because often collecting data is expensive and time consuming.

Questions you should be able to answer:

- Why is p used in the denominator of the Z-score (instead of $\hat{p}$) ?
- Why is $p = 0.35$ instead of $p = 0.4$ in H_0?
- Why is the alternative direction $>$ instead of $<$?
- Why is the `xpnorm()` in part (e) centered at 0.4 with a SD of $\sqrt{0.4 \cdot 0.6/50}$?
- Is power typically positively related to sample size? Why?