Worksheet 8 - Z hypothesis testing

Wednesday, February 18, 2026

Math 58B - Jo Hardin

Name: _____

Names of people you worked with: _____

Cat or dog?

Task: Consider the scenario from HW4 which aims to measure the proportion of all US adults who think that the extreme poverty has doubled over the last 20 years. Using the normal theory structure, perform a complete hypothesis test on whether people are randomly guessing across the three options (that is, **whether or not the true proportion** who would choose "double" **is one-third** – the one-third comes about because Rosling's survey was asked with three possible responses). Recall that 593 out of 1005 people thought that the rate of extreme poverty had doubled over the last 20 years.

The complete hypothesis test should include the following pieces:

1. Statement of the null and alternative hypotheses
2. Calculation of the test statistic (here it will be the Z score representing how many standard deviations your sample proportion is from your hypothesized value).
3. A sketch of the null sampling distribution of $\hat{p}$ with the following characteristics: center, st. dev., observed value of $\hat{p}$ all clearly labeled.
4. (Optional: Calculation of the p-value using the normal distribution.)
5. Conclusion (whether to reject the null hypothesis or fail to reject the null hypothesis and a follow-up sentence using words from the context).

To calculate the p-value, you could use the following code (with a two-sided hypothesis, the probability should be doubled). Feel free to run the code during class if you want. Otherwise, just make a note that it's provided here. Tbh, you actually don't need the exact calculation, you should be able to estimate the probability (p-value) here very closely.

```
library(mosaic)
xpnorm(your_sample_proportion,
             mean = your_hyp_val,
             sd = sqrt( your_hyp_val * (1 - your_hyp_val) / your_sample_size) )
```

Which will give you identical probabilities to the calculation done on the Z score. Try it both ways to see.

```
xpnorm(your_z_score, mean = 0, sd = 1 )
```

Hint: if the problem seems a little bit bonkers, try calculating the value of $SE(\hat{p}) = \sqrt{p \cdot (1-p)/n}$ which will give you an idea of the natural variability of $\hat{p}$ around p. After calculating $SE(\hat{p})$, do you think that 0.59 could have plausibly come from a population with $p = 1/3$?

Solution:

$H_0 : p = 1/3$
$H_A : p \neq 1/3$
where $p =$ true proportion of all US adults who believe extreme poverty has doubled over the last 20 years.

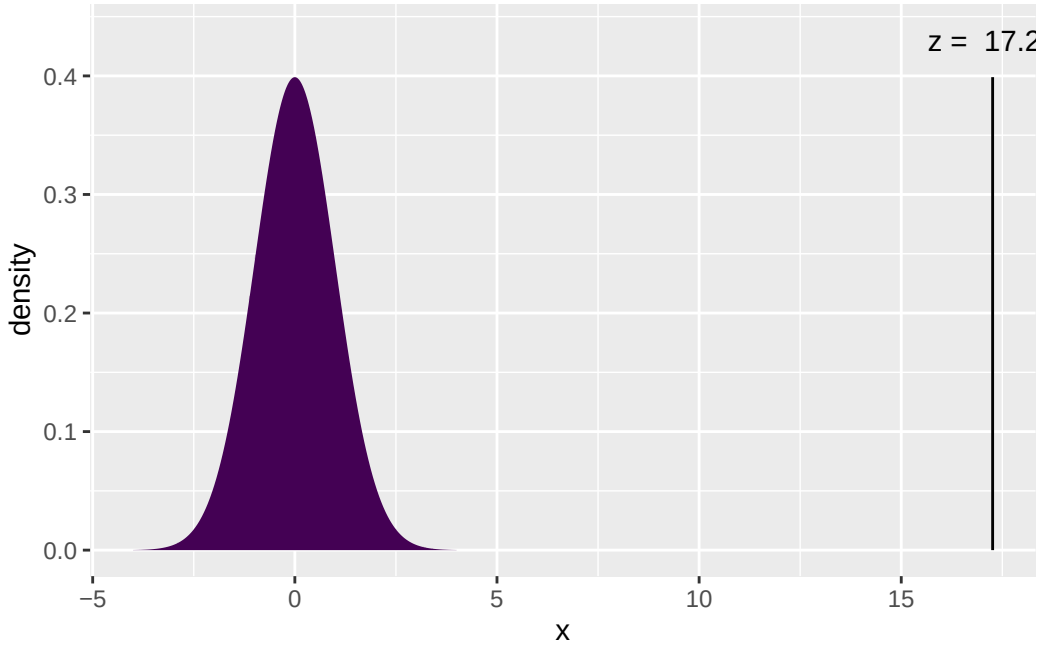
$$\begin{aligned} \hat{p} &= 593/1005 = 0.59 \\ Z &= \frac{0.59 - 0.3333}{\sqrt{0.3333 \cdot (1 - 0.3333)/1005}} = 17.26 \end{aligned}$$

p-value ≈ 0.

We reject the null hypothesis. The data collected do not come from a population of US adults who will randomly guess that the rate of extreme poverty has doubled over the past 20 years. Indeed, the true proportion of people who think that extreme poverty has doubled is much larger than 1/3.

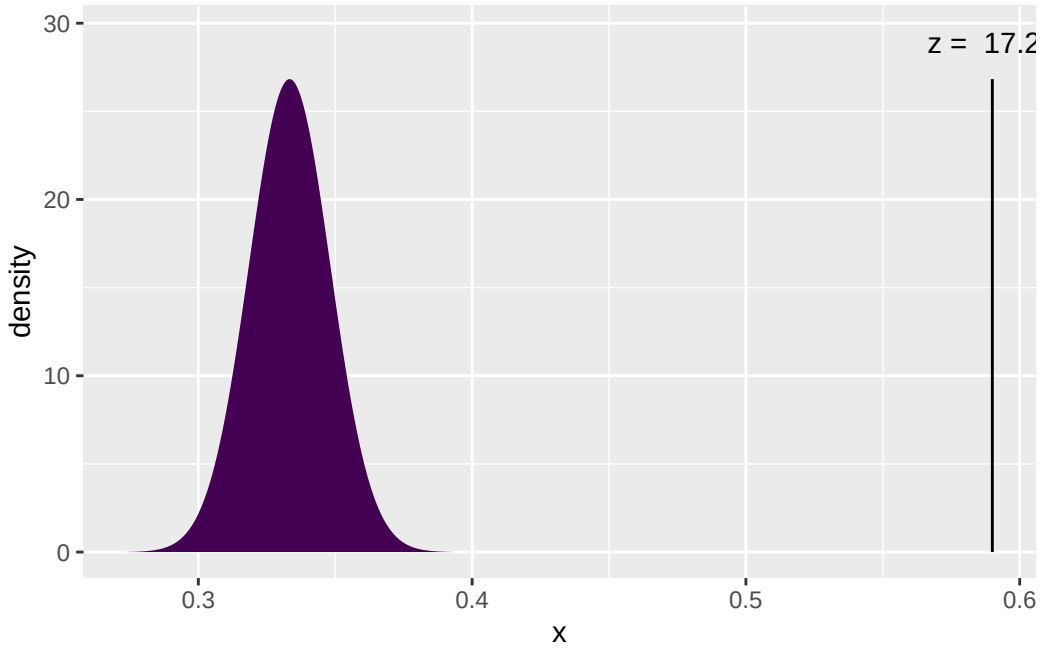
Note that in the code below, the only difference is the scale of the values on the x-axis. One of the curves is centered at zero, the other curve is centered at 1/3.

```
2 * xpnorm(17.26, mean = 0, sd = 1, lower.tail = FALSE)
```



```
[1] 9.411094e-67
```

```r
2 * xpnorm(0.59, mean = 1/3, sd = sqrt((1/3)*(2/3)/1005), lower.tail = FALSE)
```




```
[1] 9.295432e-67
```