Worksheet 12 - OR

Wednesday, March 4, 2026

Math 58B - Jo Hardin

Name: ____

Names of people you worked with: ____

Did you eat breakfast? Where? What?

Task: Helping smokers to quit is a challenging public health goal. In a study of the effectiveness of a nicotine lozenge, smokers who wanted to quit were randomly assigned to one of two groups: one group received nicotine lozenges and the other group received placebo lozenges. At the end of the 52-week study, 17.9% of the 459 subjects in the nicotine group had successfully abstained from smoking, compared to 9.6% of the 458 subjects in the placebo group.[1]

We will be working with odds ratios, using the fact that $\ln \widehat{OR}$ has a normal sampling distribution. You will find this problem easier if you start by creating a 2x2 table.

1. Using the data provided, calculate $SE(\ln \widehat{OR})$.
2. Calculate $\widehat{OR}$ for the study at hand (nicotine lozenge in numerator, placebo lozenge in denominator).
3. What if the null hypothesis is true, how likely would it be for you to observe your data or more extreme? [What does the null hypothesis being true mean?]
4. In general, how big an $\widehat{OR}$ would you need to see in order to conclude that nicotine lozenges were more effective than placebo (with a level of discernibility of $\alpha = 0.1$)?
5. a. What if the **true** OR is really 1.75 (that is, $odds_N/odds_P = 1.75$), how likely would you be to reject the null hypothesis?
   b. What if the **true** OR is really 1.75 (that is, $odds_N/odds_P = 1.75$), how likely would you be to see your data or more extreme?

[1] Example from Allan Rossman, https://askgoodquestions.blog/2021/02/01/83-better-but-not-good/}, original study by Shiffman et al. 2002, https://pubmed.ncbi.nlm.nih.gov/12038945/

Solution:

Let's start by organizing the data in tabular form:

	Nicotine lozenge	Placebo lozenge	Total
Successfully quit	82	44	126
Did not successfully quit	377	414	791
Total	459	458	917

1. $SE(\ln \widehat{OR}) = \sqrt{1/82 + 1/377 + 1/44 + 1/414} = 0.2$
2. $\widehat{OR} = \frac{82/377}{44/414} = 2.05$
3. The Z score for this setting is $Z = \frac{\ln(\widehat{OR})-0}{0.2} = \frac{\ln(2.05)-0}{0.2} = 3.589$. The relevant p-value is (notice that interest is in the right tail) 0.00017.

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1 - xpnorm(3.589)
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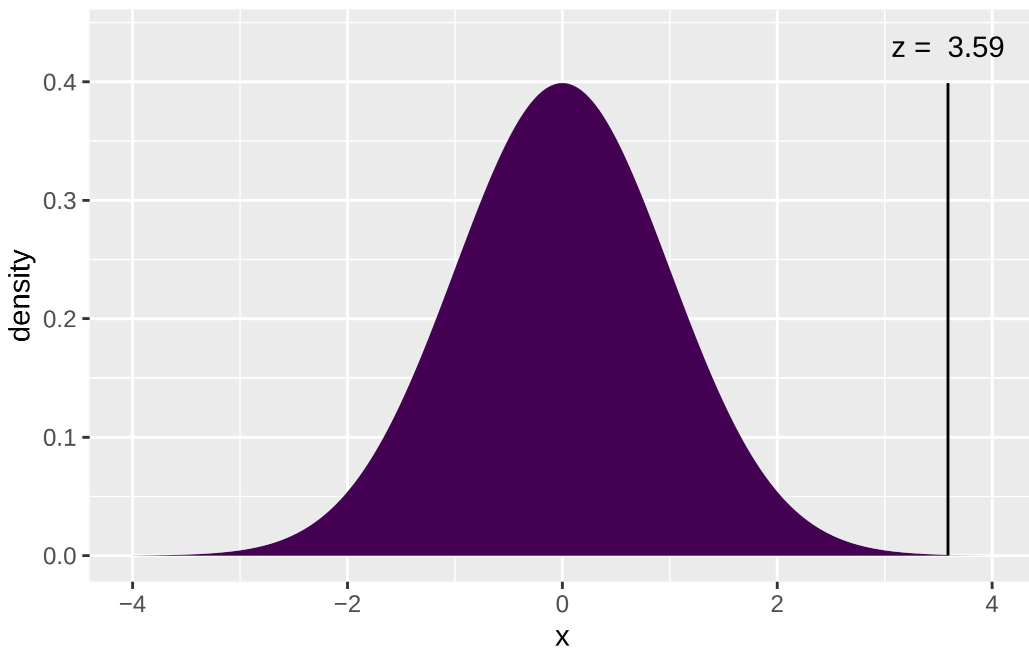


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[1] 0.0001659744
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4. In order to reject H_0, we need a Z score greater than 1.28, which translates to $\widehat{OR} > 1.29$

$$
\begin{aligned}
\frac{\ln(\widehat{OR}) - 0}{0.2} &> 1.28 \\
\ln(\widehat{OR}) &> 0.256 \\
\widehat{OR} &> e^{0.256} = 1.29
\end{aligned}
$$

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xqnorm(0.9, 0, 1)
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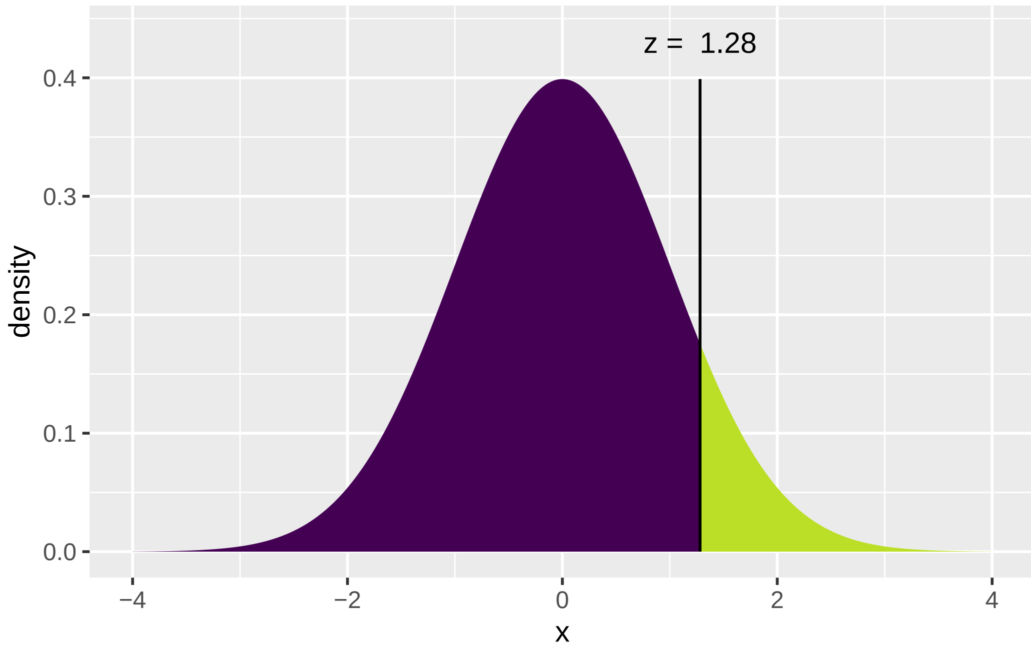


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[1] 1.281552
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5. a. What if the **true** OR was really 1.75 (that is $odds_N/odds_P = 1.5$), how likely would you be to reject the null hypothesis?

We need to calculate the probability of $\widehat{OR} > 1.29$ given that $OR = 1.75$.

$$
P(\widehat{OR} > 1.29) = P\left(\frac{\ln(\widehat{OR}) - \ln(1.75)}{0.2} > \frac{\ln(1.29) - \ln(1.75)}{0.2}\right) = P(Z > -1.525) = 0.936
$$

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1 - xpnorm(-1.525, 0, 1)
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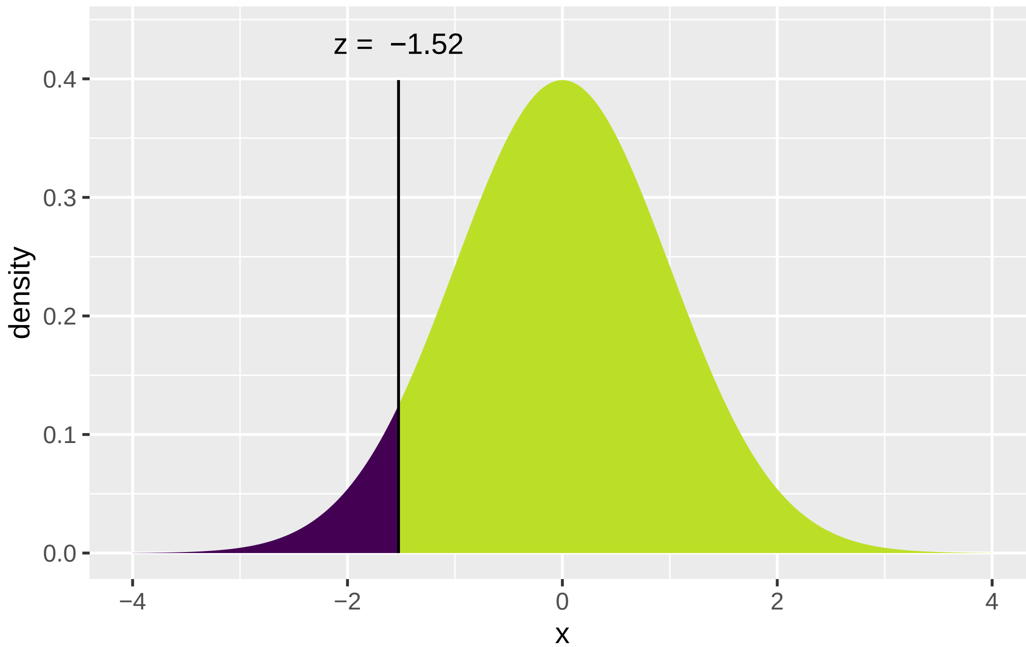


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[1] 0.9363705
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b. What if the **true** difference in proportions was really 1.75 (that is $odds_N/odds_P = 1.75$), how likely would you be to see your data or more extreme?

$$P(\widehat{OR} > 2.05) = P\left(\frac{\ln(\widehat{OR}) - \ln(1.75)}{0.2} > \frac{\ln(2.05) - \ln(1.75)}{0.2}\right) = P(Z > 0.791) = 0.214$$